Mathematics KSSM Form 2

Intensive Revision: Chapter 1 & Chapter 2 (Incl. KBAT)

— TEACHER'S COPY (SOLUTIONS) —

1. Quick Notes: The "First Principles" Approach

Chapter 1: Patterns and Sequences

- **Pattern:** A specific rule that a set of numbers follows (e.g., $+2$, $\times 3$, -5).
- **Sequence:** A list of numbers arranged in a specific order based on a pattern.
- **Fibonacci Numbers:** Start with 1, 1. Every next number is the sum of the previous two ($1, 1, 2, 3, 5, 8, 13 \ldots$).
- **Algebraic Expression (n-th term, T_n):** Think of it as a "machine". If the rule is $+3$ every time, the formula involves $3n$. E.g., for $3, 6, 9$, $T_n = 3n$.

Chapter 2: Factorisation and Algebraic Fractions

- **Expansion (Building the Area):** Multiplying out brackets. Think of it as finding the Area of a rectangle if you know its sides.
  - Single: $a(b + c) = ab + ac$
  - Double: $(a + b)(c + d) = ac + ad + bc + bd$ (FOIL method)
  - Perfect Square: $(a \pm b)^2 = a^2 \pm 2ab + b^2$
- **Factorisation (Finding the Sides):** The exact reverse of expansion. Given the Area, what were the side lengths?
  - Common Factors: $4x + 8 = 4(x + 2)$
  - Difference of Squares: $a^2 - b^2 = (a + b)(a - b)$
  - Quadratic ($ax^2 + bx + c$): Use cross-multiplication or inspection.
- **Algebraic Fractions:** They follow the *exact same rules* as primary school fractions. You MUST find a common denominator to add ($+$) or subtract ($-$).

Mastery Challenge: 50 Questions

Work quickly and accurately. Show all your workings in the spaces provided below each question.

Part A: Chapter 1 - Patterns and Sequences (25 Questions)

Determine the next two terms for the sequences:

1. $3, 7, 11, 15, \ldots, \ldots$
   Ans: $+4$ pattern $\Rightarrow 19, 23$

2. $100, 90, 80, 70, \ldots, \ldots$
   Ans: -10 pattern $\Rightarrow 60, 50$

3. $2, 4, 8, 16, \ldots, \ldots$
   Ans: $\times 2$ pattern $\Rightarrow 32, 64$

4. $144, 72, 36, 18, \ldots, \ldots$
   Ans: $\div 2$ pattern $\Rightarrow 9, 4.5$

5. $1, 4, 9, 16, 25, \ldots, \ldots$
   Ans: Perfect squares (n^2) $\Rightarrow 36, 49$

Fibonacci & Pascal's Triangle:

6. Find the next two Fibonacci numbers: $2, 2, 4, 6, 10, \ldots, \ldots$
   Ans: $4+6=10, 6+10=16, 10+16=26 \Rightarrow 16, 26$

7. Complete the sequence: $0, 1, 1, \ldots, 3, 5, \ldots$
   Ans: $1+1=2, 3+5=8 \Rightarrow 2, 8$

8. The 3rd row of Pascal's Triangle is $1, 2, 1$. What is the 4th row?
   Ans: $1, (1+2), (2+1), 1 \Rightarrow 1, 3, 3, 1$

9. What is the sum of the numbers in the 4th row of Pascal's Triangle?
   Ans: $1+3+3+1=8$

10. Find the missing number in Pascal's Triangle: $1, 4, \ldots, 4, 1$.
    Ans: Missing is $4+2=6$ (or $3+3=6$) $\Rightarrow 6$

Describe the pattern in words:

11. $4, 9, 14, 19, \ldots$
    Ans: Add 5 to the previous number.

12. $50, 45, 40, 35, \ldots$
    Ans: Subtract 5 from the previous number.

13. $3, -6, 12, -24, \ldots$
    Ans: Multiply the previous number by -2.

14. $0.1, 0.01, 0.001, \ldots$
    Ans: Divide the previous number by 10.

15. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$
    Ans: Add 2 to the denominator.

Find the n-th term (T_n) algebraic expression:

16. $4, 8, 12, 16, \ldots$
    Ans: Pattern is $+4 \Rightarrow T_n = 4n$

17. $1, 3, 5, 7, \ldots$
Ans: Pattern is $+2 \Rightarrow T_n = 2n - 1$

18. $5, 10, 15, 20, \ldots$
Ans: Pattern is $+5 \Rightarrow T_n = 5n$

19. $1^2, 2^2, 3^2, 4^2, \ldots$
Ans: Pattern is square $\Rightarrow T_n = n^2$

20. $10, 20, 30, 40, \ldots$
Ans: Pattern is $+10 \Rightarrow T_n = 10n$

Evaluate and Solve:

21. Given $T_n = 3n - 1$, find the 5th term (T_5).
Ans: $T_5 = 3(5) - 1 = 15 - 1 = 14$

22. Given $T_n = n^2 + 2$, find T_4.
Ans: $T_4 = 4^2 + 2 = 16 + 2 = 18$

23. For $5, 10, 15, 20 \ldots$, find the 20th term.
Ans: $T_n = 5n \Rightarrow T_{20} = 5(20) = 100$

24. If $T_n = 4n$, find the value of n when $T_n = 48$.
Ans: $4n = 48 \Rightarrow n = 12$

25. A sequence of triangles is made of matches. The pattern is $3, 5, 7, 9 \ldots$. How many matches are needed for the 10th triangle?
Ans: $T_n = 2n + 1 \Rightarrow T_{10} = 2(10) + 1 = 21$ matches

Part B: Chapter 2 - Factorisation and Algebraic Fractions (25 Questions)

Expand the following brackets:

26. $4(x+3)$
Ans: $4x+12$

27. $-3(2y-5)$
Ans: $-6y+15$

28. $x(x+7)$
Ans: x^2+7x

29. $3ab(a-2b)$
Ans: $3a^2b-6ab^2$

30. $-2p(4p-3q+1)$
Ans: $-8p^2+6pq-2p$

31. $(x+2)(x+4)$
Ans: $x^2+4x+2x+8=x^2+6x+8$

32. $(m-3)(m+5)$
Ans: $m^2+5m-3m-15=m^2+2m-15$

33. $(2y+1)(y-4)$
Ans: $2y^2-8y+y-4=2y^2-7y-4$

34. $(x+5)^2$
Ans: $x^2+2(5)(x)+5^2=x^2+10x+25$

35. $(3k-2)^2$
Ans: $(3k)^2-2(3k)(2)+2^2=9k^2-12k+4$

Factorise completely:

36. $6x+18$
Ans: $6(x+3)$

37. $15a^2b-10ab$
Ans: $5ab(3a-2)$

38. x^2-36
Ans: $(x+6)(x-6)$

39. $4y^2-25$
Ans: $(2y+5)(2y-5)$

40. $2m^2-18$
Ans: $2(m^2-9)=2(m+3)(m-3)$

41. $x^2+7x+10$
Ans: $(x+2)(x+5)$

42. $x^2-8x+15$
Ans: $(x-3)(x-5)$

43. $x^2+2x-24$
Ans: $(x+6)(x-4)$

44. $2x^2 + 5x + 3$

Ans: $(2x + 3)(x + 1)$

45. $3x^2 - 10x + 8$

Ans: $(3x - 4)(x - 2)$

Simplify the following algebraic fractions:

46. $\frac{3x}{7} + \frac{2x}{7}$

Ans: $\frac{5x}{7}$

47. $\frac{x}{3} - \frac{x}{4}$

Ans: $\frac{4x-3x}{12} = \frac{x}{12}$

48. $\frac{2}{5a} + \frac{1}{10a}$

Ans: $\frac{4}{10a} + \frac{1}{10a} = \frac{5}{10a} = \frac{1}{2a}$

49. $\frac{x^2-9}{x+3}$

Ans: $\frac{(x+3)(x-3)}{x+3} = x - 3$

50. $\frac{2m}{3n} \times \frac{9n^2}{4m^2}$

Ans: $\frac{18mn^2}{12m^2n} = \frac{3n}{2m}$

Part C: HOTS / KBAT Challenge (10 Questions)

These questions require you to apply your knowledge to new situations. Think from First Principles!

51. **[Real-world Sequence]** Ali saves RM 3 in the first week, RM 7 in the second week, and RM 11 in the third week, following a consistent pattern. In which week will he save exactly RM 47 for that specific week?

    Solution:
    Pattern: $+4 \Rightarrow T_n = 4n - 1$.
    $4n - 1 = 47 \Rightarrow 4n = 48 \Rightarrow n = 12$.
    Ans: Week 12.

52. **[Visual Pattern]** A pattern of regular hexagons is formed using matchsticks. The first hexagon uses 6 matches, two joined hexagons use 11 matches, and three joined hexagons use 16 matches. Formulate an algebraic expression (T_n) for this pattern and find the number of matches needed to form 20 joined hexagons.

    Solution:
    Sequence: $6, 11, 16 \cdots \Rightarrow$ Pattern is $+5$.
    $T_n = 5n + 1$.
    For $n = 20 \Rightarrow 5(20) + 1 = 101$.
    Ans: $T_n = 5n + 1$, 101 matches.

53. **[Pascal's Triangle Logic]** The sum of the numbers in the n-th row of Pascal's Triangle is given by the formula 2^{n-1} (assuming the very first tip '1' is the 1st row). Which row of Pascal's triangle has a sum of 256?

    Solution:
    $2^{n-1} = 256$
    We know $2^8 = 256$.
    Therefore, $n - 1 = 8 \Rightarrow n = 9$.
    Ans: The 9th row.

54. **[Physics Application]** A bouncing ball drops from a height of 100 cm. Each time it bounces, it reaches half of its previous height. Write down the sequence of the first 4 bounce heights. Is this an arithmetic sequence? Why or why not?

    Solution:
    Heights: $50, 25, 12.5, 6.25$.

Ans: No, it is not an arithmetic sequence. There is no common difference (it involves a constant multiplication/division ratio instead of addition/subtraction).

55. **[Fibonacci Application]** A certain species of plant grows branches in a Fibonacci sequence month by month: 1, 1, 2, 3, 5... In which month will the plant first exceed 50 branches?

    Solution:
    Sequence: $1, 1, 2, 3, 5, 8, 13, 21, 34, 55 \ldots$
    Ans: The 10th month (it will have 55 branches).

56. **[Area & Expansion]** A rectangular garden has a length of $(3x + 2)$ meters and a width of $(2x - 1)$ meters. Inside the garden, there is a square pond with sides of x meters. Write an expanded algebraic expression for the area of the grass (the garden excluding the pond).

    Solution:
    Total Area $= (3x + 2)(2x - 1) = 6x^2 - 3x + 4x - 2 = 6x^2 + x - 2$.
    Pond Area $= x \times x = x^2$.
    Grass Area $= (6x^2 + x - 2) - x^2$.
    Ans: $5x^2 + x - 2$ m^2

57. **[Perimeter & Factorisation]** The area of a rectangular whiteboard is given by the quadratic expression $(2x^2 + 11x + 12)$ cm^2. By factorising this expression, determine the length and the width of the whiteboard in terms of x. Then, find its perimeter in terms of x.

    Solution:
    Area $= 2x^2 + 11x + 12 = (2x + 3)(x + 4)$.
    Length and Width are $(2x + 3)$ and $(x + 4)$.
    Perimeter $= 2(L + W) = 2(2x + 3 + x + 4) = 2(3x + 7)$.
    Ans: Length/Width: $(2x + 3)$ & $(x + 4)$. Perimeter: $6x + 14$ cm.

58. **[Volume & Factorisation]** A 3D printed rectangular storage box has a volume of

$(x^3 - 9x)$ cm^3. If its height is x cm, find the expressions for its length and width by factorising completely.

Solution:
Base Area $= \frac{\text{Volume}}{\text{Height}} = \frac{x^3-9x}{x} = x^2 - 9$.
Factorising Difference of Squares: $x^2 - 9 = (x + 3)(x - 3)$.
Ans: The length and width are $(x + 3)$ cm and $(x - 3)$ cm.

59. **[Algebraic Fractions - Speed]** A remote-controlled car travels a distance of $(x^2 - 16)$ meters in $(x - 4)$ seconds. Find the simplified algebraic expression for the speed of the car. If $x = 10$, what is the actual speed of the car in m/s?

Solution:
Speed $= \frac{\text{Distance}}{\text{Time}} = \frac{x^2-16}{x-4}$.
Simplify: $\frac{(x-4)(x+4)}{x-4} = x + 4$.
If $x = 10$, Speed $= 10 + 4 = 14$.
Ans: Simplified speed is $(x + 4)$ m/s. Actual speed is 14 m/s.

60. **[First Principles Logic]** Simplify the expression: $\frac{1}{x-3} - \frac{6}{x^2-9}$. *Bonus:* Explain logically why x cannot be equal to 3 or -3 in this mathematical scenario.

Solution:
$\frac{1}{x-3} - \frac{6}{(x-3)(x+3)} = \frac{(x+3)}{(x-3)(x+3)} - \frac{6}{(x-3)(x+3)} = \frac{x-3}{(x-3)(x+3)}$.
Simplify to get $\frac{1}{x+3}$.
Ans: $\frac{1}{x+3}$.
Bonus Explanation: x cannot be 3 or -3 because it would make the denominator zero ($3-3=0$ or $(-3)^2-9=0$). In mathematics, dividing by zero is undefined (Math Error).

— END OF REVISION —

"A problem well-stated is a problem half-solved." - Charles Kettering